

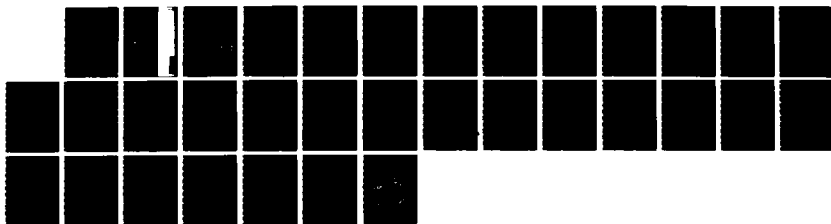
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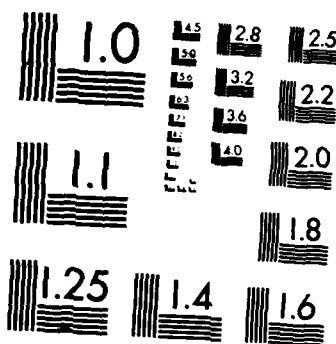
CONSTRAINED STOCHASTIC NETWORK MODELS FOR OPTIMAL  
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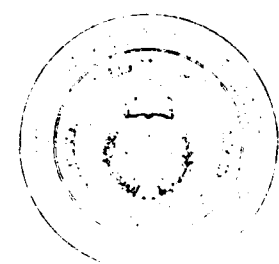
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Research Report CCS 514

CONSTRAINED STOCHASTIC NETWORK MODELS  
FOR OPTIMAL DESIGN MODIFICATIONS

by

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B. Golany  
J. Masters\*\*

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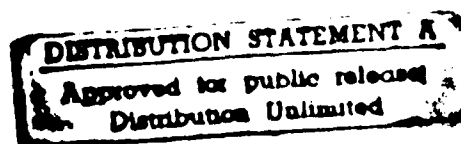
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\*\*U.S. Air Force

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CONSTRAINED STOCHASTIC NETWORK MODELS FOR  
OPTIMAL DESIGN MODIFICATIONS

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A. Charnes, W.W. Cooper, B. Golany, J. Masters

ABSTRACT

A new class of constrained stochastic network models is formulated in a manner that can be used to support a variety of analyses of optimal design modifications for special weapons systems under budgetary and other constraints. Component modifications to the B52G system are considered in a prototype example.

KEY WORDS

Weapons Systems

Optimal Design Modification

Networks

Stochastic



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## 1. Background

Over the past three decades, the evolving complexity of aerospace technology has generated a significant problem for decision-makers in the United States Air Force. Weapon systems are large, complex, expensive, and not perfectly reliable. Observed unreliabilities of sub-systems and components can be expected to degrade the wartime mission effectiveness of aircraft systems significantly. Most, if not all, subsystems and components can be made more reliable through "modification" processes involving the redesign, reengineering, and retrofitting of improved components and structures. However, modification is typically an expensive procedure. For example, the Avionics Modernization Program for F/FB111 series of fighter/bombers, which will update its 1960's era analogue processors to 1980's digital technology, will cost \$1.1 billion. Re-engining of the KC-135 refueling aircraft will cost over \$7 billion.

Generally speaking, it is far less expensive to update an existing weapon system than to field an entirely new replacement. From a managerial perspective, this leads to the following problem: From the very large set of all possible modifications to a weapon system, which subset should be chosen? In principle one should choose the subset which, for any arbitrary budget constraint, will generate the maximum increase in weapon system combat performance.

The principle is simple but the problem is very complex. Analytic formulations and aids to solution are presently lacking and so the problem is currently being addressed with subjective ad-hoc procedures. This paper seeks to remedy this situation by describing a possible approach to analytic

treatment of this decision process that can lead to selection of cost-optimal sets of modifications from the options available for a given weapon system.

Many models and techniques have been used by the U.S. Air Force (typically in the design phase) to evaluate various combinations of subsystem and component reliabilities and their effects on weapon system reliabilities and expected mission effectiveness. For example, the "Damage Expectancy" Model developed at USAF Strategic Air Command can be used to show how changes in component reliabilities may be expected to affect the ability of SAC's strategic bomber fleet to execute its assigned wartime tasking successfully. However, this model is purely descriptive. It does not identify which components ought to be selected for modification, and it does not deal with the costs of effecting modifications. Models for effecting optimal choices in other areas are also available. Indeed, USAF has a long history of developing and using component-level optimization procedures such as METRIC [9], Mod-METRIC [8], Dyna-METRIC [6], and so forth. However, these models optimize weapon system availability (as opposed to mission effectiveness) by augmenting spare parts stockage. Component unreliabilities, or failure rates, are accepted at their current performance levels. These models do not address reliability improvements as decision variables and, as a consequence, the solution to high failure rate problems may often take the form of increased stocking of particular items.

The models and methods discussed in this paper are addressed to altering the performance of a total system with respect to ranges of possible missions. Although discussed mainly in terms of a particular weapon system, the approach suggested here can be generalized to many other systems and contexts.

Generically the missions to be discussed in this paper are restricted to single aircraft systems. In particular we are concerned with launch from a base in the continental United States to deliver nuclear ordnance to a predetermined target at a distance of thousands of miles. A strategic bomber, such as the B-52, the FB111, or the B1-B is a large, heavy, complex and expensive system outfitted with electronic subsystems to provide communications, navigation, target acquisition and weapon release, electronic counter measures such as radar jamming, signal generators and chaff dispensers, low-level terrain following radar, and so forth. Increasing sophistication of enemy defensive systems, such as improved radar systems, surface-to-air missiles, and interceptor aircraft, has made the strategic bomber mission more challenging and has led to more complex technological approaches to strategic bomber design.

To make matters even more complex the mission profile of the strategic bomber is usually lengthy, and not all components or subsystems are equally critical during all stages of the mission. Thus, not all subsystem reliabilities are equally important.

A typical B52 mission profile might be described as follows:

1. Pre-launch. Given suitable warning, a large percentage of the fleet would be made ready--armed, fueled, and standing by for launch. Given minimum warning, only those aircraft standing "alert" would be fully ready. Others would be in maintenance, flying training missions, and so forth.
2. Launch. The aircraft must achieve engine start, etc., and become airborne rapidly to avoid attacking forces. Aircraft might disperse to remote sites prior to launch. Under wartime conditions, even seriously degraded aircraft (i.e., aircraft with many inoperative subsystems) would be launched rather than be left to relatively certain destruction at their bases.



3. Cruise. During this stage the aircraft would fly at high altitude over thousands of miles of undefended territory. The aircraft would engage in one or more aerial refueling operations.
4. Penetration. During this stage the aircraft must traverse hundreds or thousands of miles of heavily defended enemy territory. The aircraft will fly at an extremely low altitude and, in some cases, at supersonic speed, relying on precise navigation, radar and electro-optical terrain following systems, radar jamming, decoy missiles, defense suppression weapons, and other tactics to avoid enemy defenses.
5. Weapon Release. At one or more points, the aircraft would release nuclear ordnance in the form of either gravity bombs or stand-off air launched cruise missiles. The effectiveness of those weapons will depend upon the ability to stay on course and on schedule, on the condition of the aircraft at time of weapon release, and on the ability of the aircraft and crew to achieve the designed accuracy of the delivery system.
6. Recovery. The aircraft would exit the target area, continue to avoid enemy defensive measures, and recover at a base in the U.S. or elsewhere.

As an aircraft progresses through its mission, component and subsystem failures will occur (due to component unreliabilities) which will affect the ability of the aircraft to complete each mission stage successfully. As the condition of the aircraft degrades, probabilities of successful mission stage completion will be reduced. It is this process that we shall attempt to portray by means of network models that can accommodate the kinds of complexities involved and which can be given computationally implementable form to achieve the optimizations that are desired.

## 2. Model Characteristics

The model to be developed varies considerably from the event tree framework conditional chance-constrained programming model of [7], the stochastic networks in [2] and in the reliability literature. Past work there as cited in [10], involves extremely sophisticated arguments on very special stochastic networks (e.g., all branches having equal reliabilities) and establishes properties of NP completeness and, at best, heuristics for solving examples. No models are developed which will provide analytical characterization of the problem which analytically provides economic trade-offs for changes in reliabilities or gains in mission achievement associated with such.

We develop a type of aggregated "pseudo-deterministic" model which we reduce to one of convex programming type which can therefore provide evaluations and can be conveniently solved with any of a number of existing methods and software.

The basic element in the model is the replacement of the stochastic variable on an arc between two nodes (corresponding to different states) by a simple aggregate probability of transfer based on theoretically and operationally estimated reliability of subsystems involved in effecting the transition between states. Thereby a "pseudo-deterministic" model is achieved. As detailed in the paper, the model achieved is one of directed network type with additional side conditions of a nonlinear variety but which define convex sets.

Our development can be motivated as follows: The myriads of weapon system components are first related to weapon subsystems which in turn are related to the various mission stages in which they operate as interdependent units. In this way the criticality of component failures can be related to their possible impact on the mission and mission stages through their subsystems and assemblies of subsystems.

A stochastic "flow" network which will enable effective representation and computation of expected values for any mission can be developed in the following manner. To initiate the process a unit of flow is entered at the left-most node of the network. Transition probabilities in terms of continuation or degradation are then prescribed for arcs leading from this node to the next stage in the mission. Additional arcs and nodes are then similarly used to portray the possibilities for transiting to further stages in the mission.

In contradistinction to usual network forms and uses, the flows on the arcs of this network will represent cumulative probabilities of transiting the stage associated with each arc. These values correspond to prescribed proportions (or probabilities) of branching flow from the entering node. For example the initial flow from the left-most node is thus broken up by branching proportions (or probabilities) that sum to unity--the value of the entering flow that initiated the process.

The usual network condition requires that the sum of the entering flows must equal the sum of the departing flows at each node in the network. Here we have added the condition that the branching departures are to be in specified proportions. These proportions then correspond to the probabilities of transiting from the condition represented by the entering node and these probabilities are determined from the reliabilities of the set of components that may be critical at that stage.

Components and subsystems reliabilities may be related to these transition probabilities in many ways that depend on the manner in which they are aggregated. In this paper we shall proceed with two examples to illustrate how the status of the aircraft in these may be characterized in terms of five selected aircraft conditions at each stage.

We first do this in graphic form as in Figure 1. Under each of the stages listed at the top of the Figure will be found a set of arcs that correspond to the possibilities at that stage. The possible modes of continuation from each stage are also similarly represented. For instance in the stage under Launch the five arcs starting at the top represent, respectively, continuation with no degradation, light degradation, severe degradation, return to base, and loss of the aircraft.

In this example no upward transition from a more serious to a less serious state of degradation is permitted although this, too, can be added when warranted. Also after attainment of the nodes listed under Damage (to Target) all arcs lead to Return to Base.

As already mentioned the probabilities with which these events occur will be determined by the reliabilities of the systems that are critical in each state (named on the left) and each stage (named at the top) of Figure 1.

Apparently our network can be extended and elaborated. It can also be contracted or aggregated for various analytical purposes. For instance, one might use a much simpler network of agglomerated arcs, one for each of the seven stages listed at the top of Figure 1. These could then be detached and analyzed for their network effects and/or recombined according to the probabilities of different behaviors at each stage.

# STAGE-TRANSITION DIAGRAM

Stage A/C Status	Pre- Launch	Launch	Flight over Friendly Territory	Penetration	Flight over Hostile Territory	Release	Damage	Return
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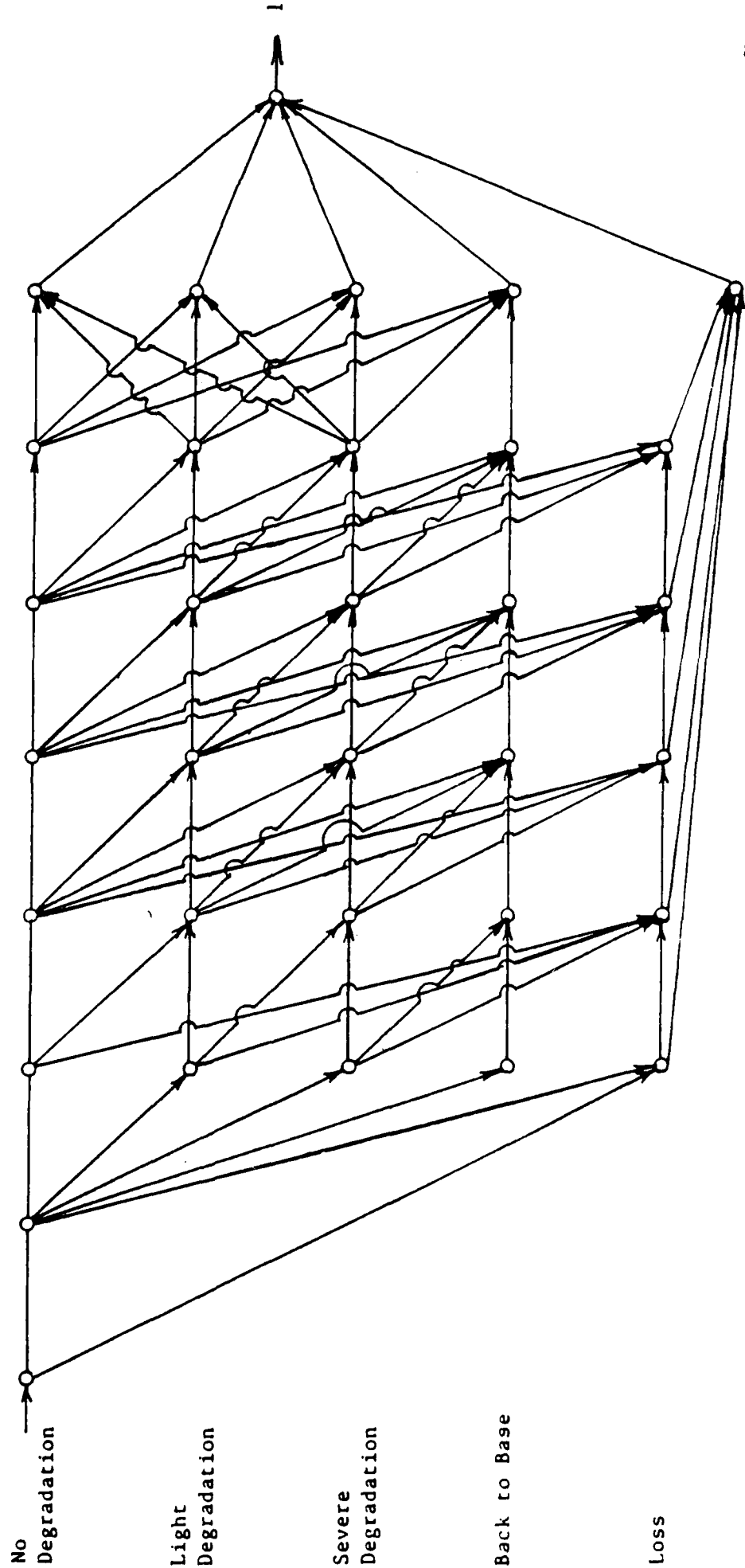


FIGURE 1

Thus, in summary, we may conceptualize in terms of an aircraft being entered in the left-most node and either continuing without degradation or experiencing degradation with the probabilities that are pertinent at each stage. As successive nodes are reached portions of the stochastic process are again applied and the process continues until the stage of Weapons Release (or Abort) is achieved. Movement to the next set of nodes on the right of Figure 1 then reflects differing states of Damage to the Target that are achievable with probabilities that correspond to the status of the aircraft at this stage. Finally, the last stage noted at the top of Figure 1 represents either Return to Base or complete loss of the aircraft.

### 3. Cost and Budget Constraints

In principle the probabilities of transition (or proportions) for the branching flows from a node are known from data on the current operational status of an aircraft. For the purpose of selecting cost optimal sets of modifications we need to extend the previously described modelling process to allow for possible changes in these probabilities that can be effected from modifications that will be additional decision variables. The latter in turn need to be related to the costs of effecting these changes in the probabilities.

The cost of increasing the probability of transiting a stage without degradation will generally assume the form of a convex function of the increments in probability in a manner such as is depicted in Figure 2. In the analytical model that we employ such costs are entered into a budgetary constraint on modification cost which will form part of a convex programming problem. In particular, we shall formulate our "system design

modification planning model" so that it assumes the form of a nonlinear programming problem for maximizing an expected damage measure.

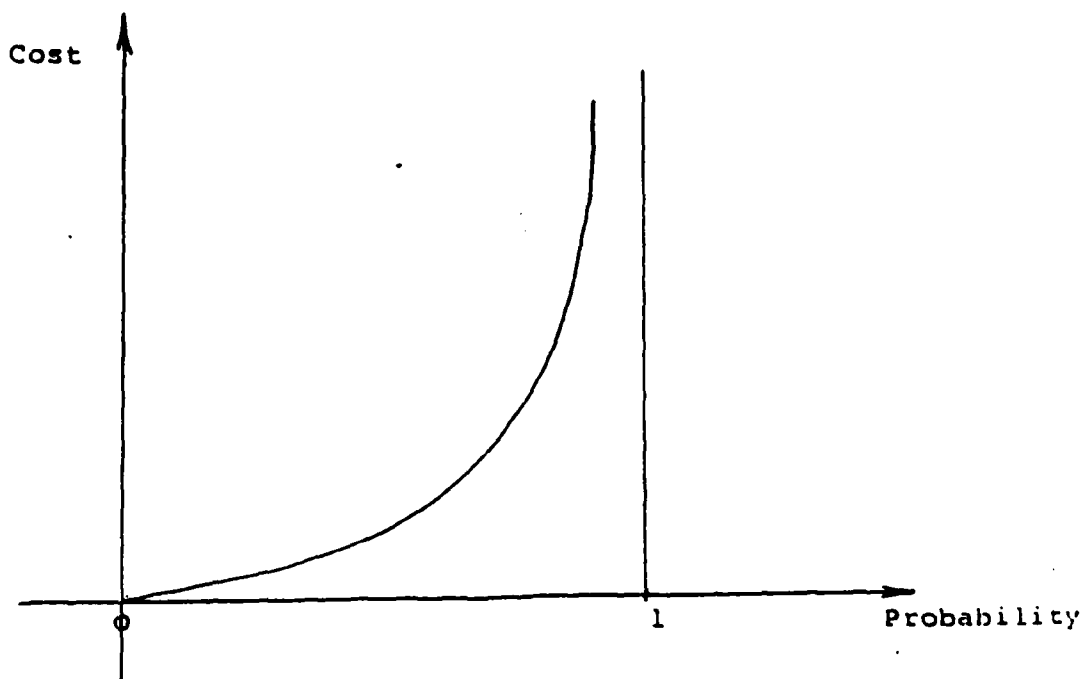


Figure 2

We begin by introducing the following notation and definitions:

#### I. Sets

$N = 1, 2, \dots, 1, \dots, m$  list of nodes.

$A = 1, 2, \dots, k, \dots, n$  list of arcs.

$O_i$  = set of arcs originating at node  $i$ .

$T_i$  = set of arcs terminating at node  $i$ .

$W$  = set of arcs "weighted" by damage scores.

$M$  = set of arcs where modifications are suggested.

#### II. Arcs and Nodes

$s$  = source node.

$t$  = terminal node.

$l_k$  = current reliability, lower limit on reliability measure  
at arc  $k$ .

$u_k$  = upper limit on potential reliability improvement at arc  $k$ .

$w_k$  = damage measure for arc  $k$  in set  $W$ .

$C_k(y_k)$  = modification cost incurred for achieving probability  $y_k$  on  
arc  $k$  in  $M$ .

### III. Decision Variables

$x_k$  = flow on arc  $k$

$y_k$  = probability on arc  $k$ ,  $(k \in O_i)$  = proportion of inflow to node  $i$   
which is constrained to flow through arc  $k$ .

### IV. Budget Information

$B$  = total budget to be allocated for modifications.

Our model might now be stated mathematically as the following nonlinear  
constrained network programming problem:

$$(1) \quad \text{Max} \quad \sum_{k \in W} w_k x_k$$

s.t.

$$(2) \quad \sum_{k \in M} C_k(y_k) \leq B \quad (\text{budget})$$

$$(3) \quad \sum_{k \in O_i} x_k - \sum_{r \in T_i} x_r = 0, \quad i \in N, \quad i \neq s, t \quad (\text{flow conservation at nodes})$$

$$(4) \quad y_p \sum_{k \in T_i} x_k \geq x_p, \quad p \in O_i, \quad i \in N \quad (\text{proportionality branching})$$

$$(5) \quad \sum_{j \in T_t} x_j = 1 \quad (\text{demand flow conservation})$$

$$(6) \quad \sum_{j \in O_s} x_j = 1 \quad (\text{supply flow conservation})$$

$$(7) \quad l_k \leq y_k \leq u_k, \quad k \in A \quad (\text{enhancement bounds})$$



$$(8) \quad x_k \geq 0, k \in A \quad (\text{uni-directional flow})$$

Notice that (7) is given in general for all the arcs in the network. When there is no modification to be suggested for arc  $k$ ,  $u_k = 1_k$ . Conditions (5) and (6) imply that one aircraft "enters" the network through the source node and "leaves" it through the terminal node. The problem of nonlinearity arises from the budget constraint and from using a product form inequality in (4) to obtain a convex program with the required proportionate branching.

For the proportionality branching we are using, the inequalities in (4) need to be supplemented by another set of conditions (4a, below) by virtue of the following considerations. Suppose that a modification enhances the probability of transition across some arc. This in turn will also affect the probabilities of transition in degraded conditions at this same stage. Since the transitions are mutually exclusive and collectively exhaustive their new probabilities must add to unity in a manner that accommodates the new probability on the enhanced arc.

How these probabilities will change depends, of course, on the nature of modifications introduced into the system. The pattern of such changes requires future research attention but a plausible approximation might specify merely that the values of the new degradation probabilities are reduced according to a consistency with probabilities adding to unity.

We now show that adjoining these conditions will then guarantee that the inequalities (4) are satisfied as equalities as is required. Also, the degradation arcs emanating from the node from which the enhanced arc originates will have their probabilities reduced in proportion to their entering values.

Suppose we add

$$(4a) \quad \sum_{x_p \in O_i} y_p = 1$$

Now if  $y_p \sum_{x_k \in T_i} x_k > x_p$  for some  $x_p \in O_i$ ,

$$\text{Then } \sum_{x_p \in O_i} y_p \left( \sum_{x_k \in T_i} x_k \right) > \sum_{x_p \in O_i} x_p$$

$$\text{and } \sum_{x_p \in O_i} y_p > \left( \sum_{x_k \in T_i} x_k \right) / \sum_{x_p \in O_i} x_p = 1, \text{ by (3)}$$

But by (4a) we have  $1 > 1$ , a contradiction.

Hence  $x_p = y_p \sum_{x_k \in T_i} x_k$  for all  $x_p$  as required. Evidently, increase in  $y_p$

decreases the  $x_k \in T_i$  and in proportion to the entering value  $x_p$ .

We now turn to linear approximation of (4),

$$y_p z^i \geq x_p, \text{ where}$$

$$z^i = \sum_{j \in T_i} x_j$$

Although  $y_p z^i$  is not a concave function, the  $(y_p, z^i, x_p)$  set is convex.

For fixed  $x_p$ , this is the epigraph of a hyperbola in  $(y_p, z^i)$  which is the intersection of the half-spaces of its supporting hyperplanes. At any given point on such a hyperbola (denoted by  $\bar{y}_p, \bar{z}^i$ ) the half space defined by the supporting hyperplane (in this case a line) is given by the inequality (after omitting the super- and sub-scripts):

$$(4b) \quad \bar{y}z + \bar{z}y \geq 2\bar{x}$$

where  $\bar{x}$  is the fixed  $x_p$ .

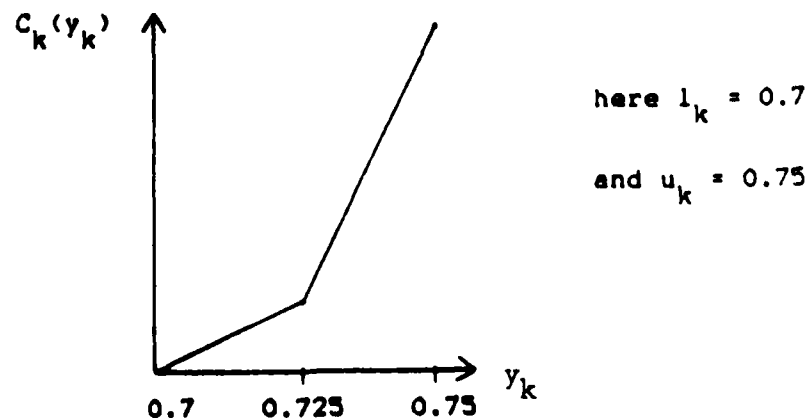
We have presented the above model in this manner to exhibit some of the computational expedients that may be employed. Clearly if we run the model

without (4a) we can obtain an upper bound, or over-optimistic estimate of the maximum expected damage score. We may refer to this as the "unconstrained allocation of flow" to unmodified arcs and contrast it with the "proportional" allocation of flow assigned to these same arcs when (4a) is employed. For purposes of sensitivity comparison we present an example calculated both ways in the section that follows.

### Example

The relations between costs and alterations in probability represented in Figure 2 are best regarded as idealizations. It will at best be generally possible to obtain something in the way of estimates over limited ranges of possibilities that may be represented in the manner of Figure 3 where  $l_k = 0.7$  represents a lower limit for some component (or system of components) established from prior experience and  $u_k = 0.75$  represents an upper limit for what is attainable from design changes. With further refinement, changes in probability-cost relations may be found to occur in the manner of "semivariable" costs, as they are called in the literature of accounting [5].

This is the kind of behavior in Figure 3 where the slope of the modification cost curve changes at 0.725 from a lower to a higher value.



For the present illustrative example we shall simplify matters even further and assume that all such cost curves consist of only one linear segment that is operative between a lower bound,  $l_k$ , and an upper bound,  $u_k$  for the arcs  $M$  in which enhancements are to be made. Note that for arcs where no enhancements are to be considered we will leave  $u_k = l_k$  where the latter represents the present reliability value for the associated arc.

The example we shall use consists of a network with 27 nodes and 66 arcs. The pertinent arc information is given in Table 1 and the network is depicted in Figure 4. In the notation we are using we have

$$N = \{1, 2, \dots, 27\}, s = \{1, t = 27\}$$

i.e., we have 27 nodes with node 1 distinguished as the source node and node 27 as the terminal node. We also list the arcs on which modifications might be made as

$$M = \{2, 7, 21, 35, 49\}$$

See Table 1 for the associated costs, under  $c_k$ , and the upper and lower limits under  $l_k$  and  $u_k$ , respectively.

The orientation is toward maximizing the total expected damage scores as given by the values assigned to the arcs

$$W = \{63, 64, 65, 66\}.$$

These are referred to as "goal arcs" in Table 1 to which the weight values listed there represent the  $w_R$  assigned in the function being maximized in

(1).<sup>1</sup>

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<sup>1</sup> See Charnes, Cooper Lewis and Niehaus [3] where this goal arc concept was first introduced and see Charnes, Cooper, Nelson, and Niehaus [4] for its further developments and extensions.

To make all of the modifications a budget of 305 (in millions of dollars) would be needed but we are limited to a budget of B = 140.

FIGURE 4

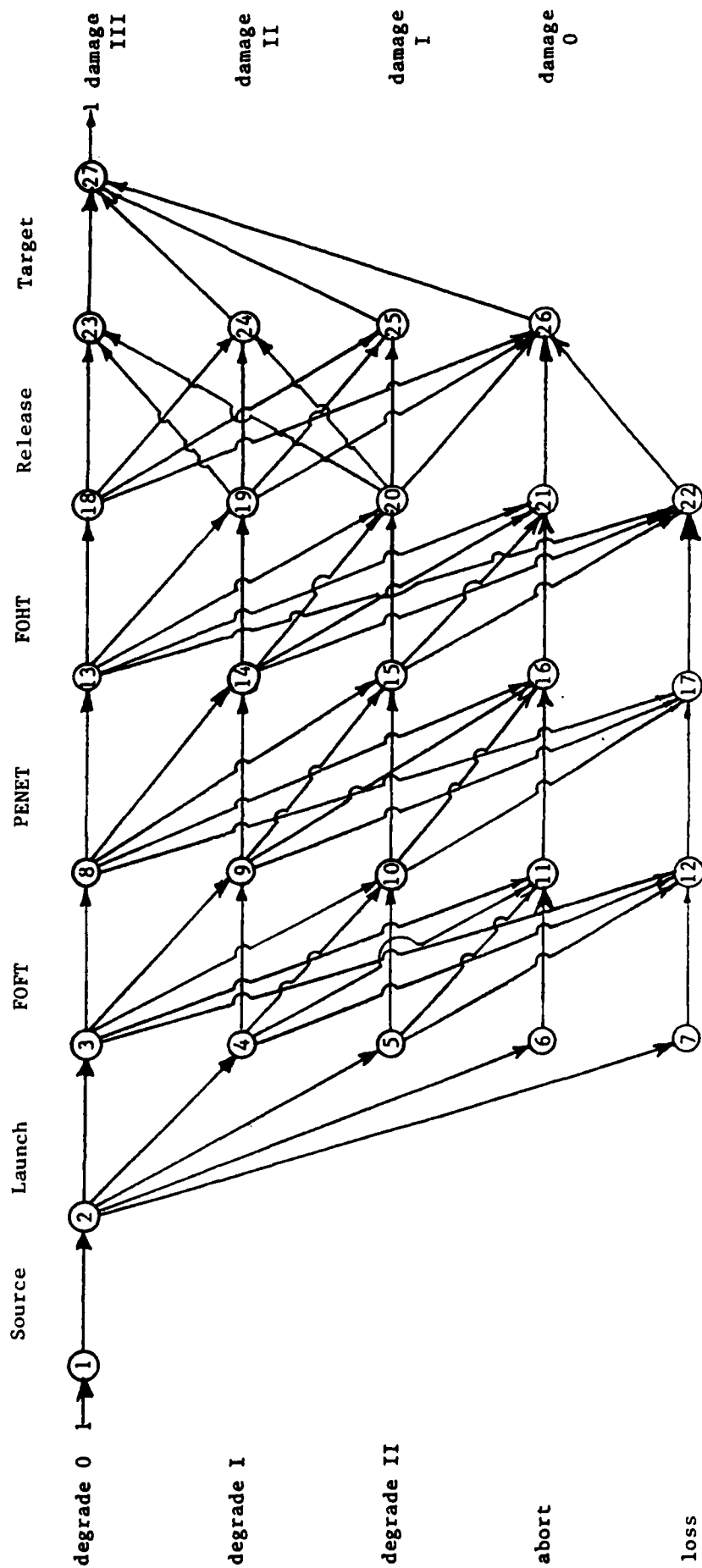


Table 1

Arc #	from node	to node	$l_k$	$u_k$	$c_k$
1	1	2	1.0		
2	2	3	.8	.9	50
3	2	4	.05		
4	2	5	.05		
5	2	6	.05		
6	2	7	.05		
7	3	8	.8	.9	100
8	3	9	.05		
9	3	10	.05		
10	3	11	.05		
11	3	12	.05		
12	4	9	.85		
13	4	10	.05		
14	4	11	.05		
15	4	12	.05		
16	5	10	.9		
17	5	11	.05		
18	5	12	.05		
19	6	11	1.0		
20	7	12	1.0		
21	8	13	.6	.8	75
22	8	14	.2		
23	8	15	.1		
24	8	16	.05		

25	8	17	.05		
26	9	14	.8		
27	9	15	.1		
28	9	16	.05		
29	9	17	.05		
30	10	15	.8		
31	10	16	.1		
32	10	17	.1		
33	11	16	1.0		
34	12	17	1.0		
35	13	18	.8	.9	60
36	13	19	.05		
37	13	20	.05		
38	13	21	.05		
39	13	22	.05		
40	14	19	.75		
41	14	20	.1		
42	14	21	.1		
43	14	22	.05		
44	15	20	.8		
45	15	21	.1		
46	15	22	.1		
47	16	21	1.0		
48	17	22	1.0		
49	18	23	.8	.85	20



50	18	24	.1	
51	18	25	.05	
52	18	26	.05	
53	19	23	.1	
54	19	24	.7	
55	19	25	.1	
56	19	26	.1	
57	20	23	.1	
58	20	24	.1	
59	20	25	.6	
60	20	26	.2	
61	21	26	1.0	
62	22	26	1.0	<u>weights on goal arcs</u>
63	23	27	1.0	50
64	24	27	1.0	30
65	25	27	1.0	10
66	26	27	1.0	0

### 5. Solutions

To gain some perspective on the solutions to follow we start with Figure 5. In this case no modifications can be made so that all values are held at their lower bounds,  $l_k$ , in Table 1. Proceeding in the manner previously indicated the unit flow entered at the source node on the left in Figure 5 is decomposed into the ratios shown flowing out of node 2. No modification is made so the probability of transition from node 2 to node 3 remains at 0.8. Since no modification is made, the probability of reaching node 8 (see Table 1) is  $0.8 \times 0.8 = .64$  with the difference between 0.8 and 0.64 being represented by the degrading probabilities (or proportions) of 0.04 each assigned to the other arcs leading out of node 3. Moving to the next arc the just attained value of 0.64 is next multiplied by 0.6 to obtain the 0.384 value shown on the arc connecting node 8 to node 13. This value is in turn multiplied by 0.8 to obtain the value 0.3072 assigned to the arc from 13 to 18 in Figure 5--which corresponds to arc number 35 in Table 1--and the value of 0.2476 on the arc connecting nodes 18 and 23 is similarly derived by multiplying 0.3072 by 0.8.

At this point in Figure 5 it will be observed that arcs are encountered flowing upward which is to say that all of these arcs correspond to activities associated with Release and Target attainment activities. Note that flows into node 23 under Release also occur from nodes 19 and 20. Adding the probabilities on these arcs to the just computed probability for Release on the arc from nodes 18 to 23 we obtain

$$0.24576 + 0.01647 + 0.01524 = 0.2774$$

which is the probability of Target attainment for the arc going from node 23 to node 27. To get the expected damage score we also need to consider the probabilities of target damage from degraded states such as the flows going

to node 27 from nodes 24, 25 and 26 with their associated probabilities. The "weights on goal arcs" given at the end of Table 1 reflect the damage assessments from each of these flow possibilities and so we obtain the expected damage score via

$$0.2774 \times 50 = 13.870$$

$$0.1612 \times 30 = 4.836$$

$$0.1232 \times 10 = 1.232$$

$$0.4380 \times 0 = 0.0$$

$$\text{Total:} \quad 19.938$$

The probabilities on the other arcs in Figure 5 are similarly derived from the data of Table 1, and hence need not be discussed in detail. In any event the result for Figure 5 may be compared with the solution in Figure 6, where the latter exhibits the solution that is optimal for the unconstrained allocation, i.e., the allocation with  $\sum_p y_p = 1$  omitted. Thus optimal

allocation of the budget value of  $B = 140$  (million dollars) raises the score from 19.38 to 27.305, an increase of some 40% in the expected damage score.

To correct for this somewhat optimistic estimate we introduce the conditions associated with (4a) and achieve the constrained proportional allocation shown in Figure 7. Although the resulting value of 25.095 is lower than what was achieved in the program for Figure 6, it is still a substantial increase over the program exhibited in Figure 5.

FIGURE 5  
SOLUTION WITH NO MODIFICATIONS

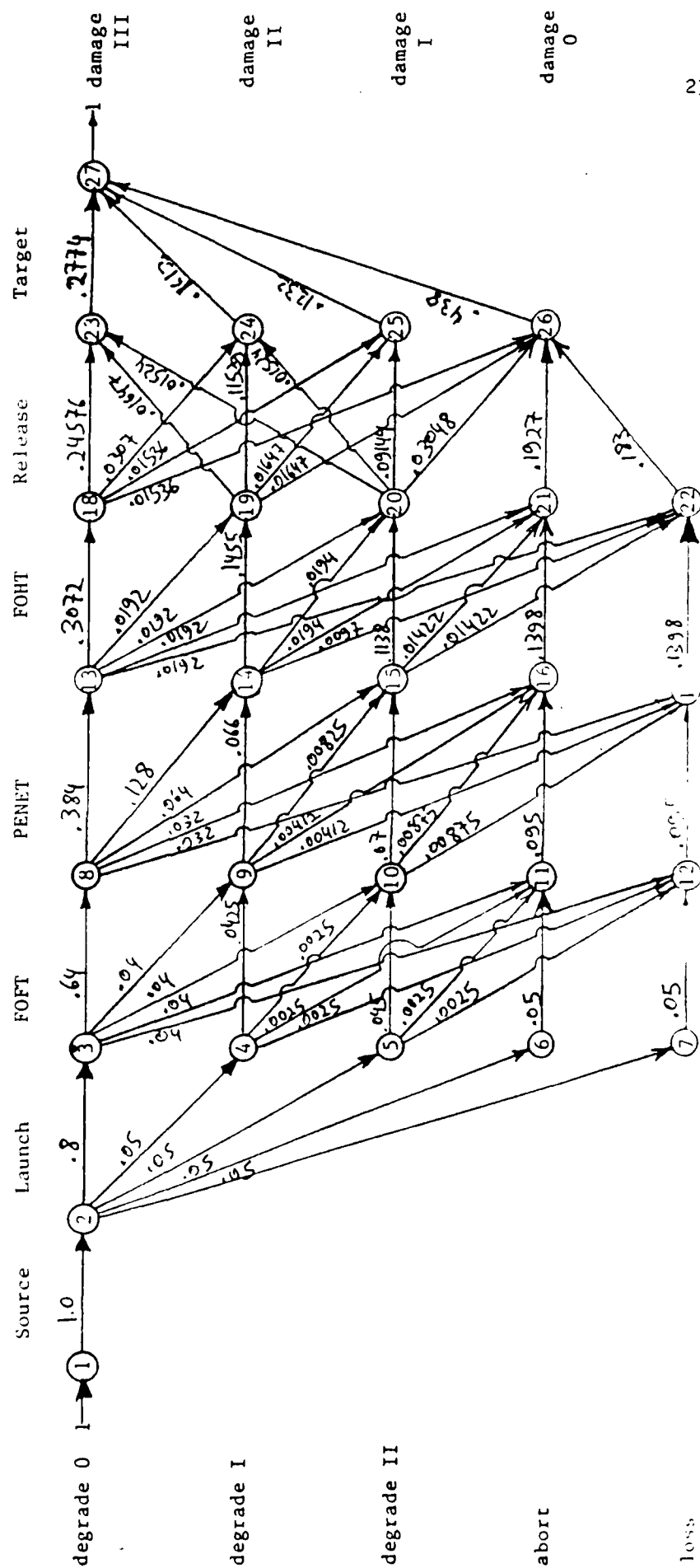


FIGURE 6  
UNCONSTRAINED ALLOCATION

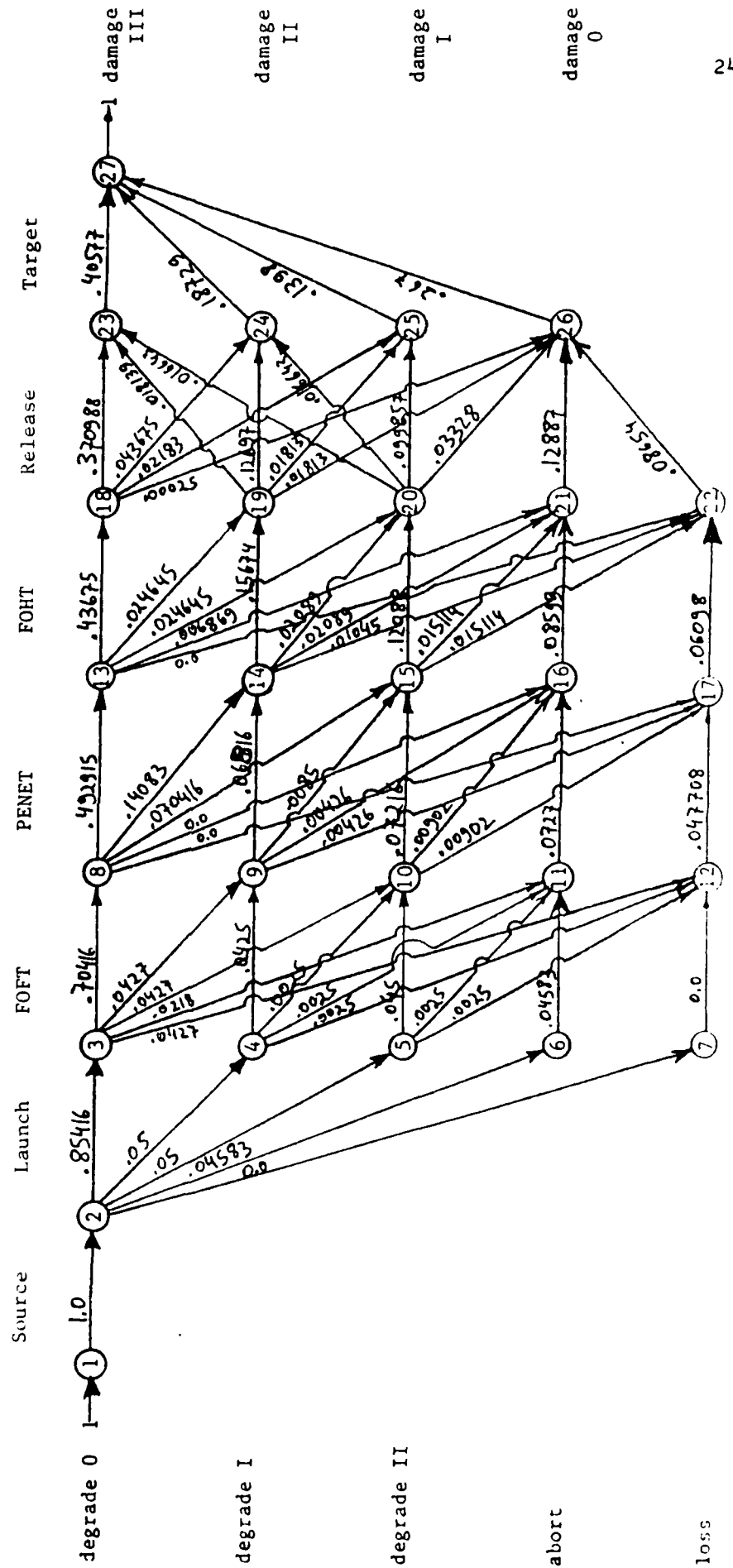
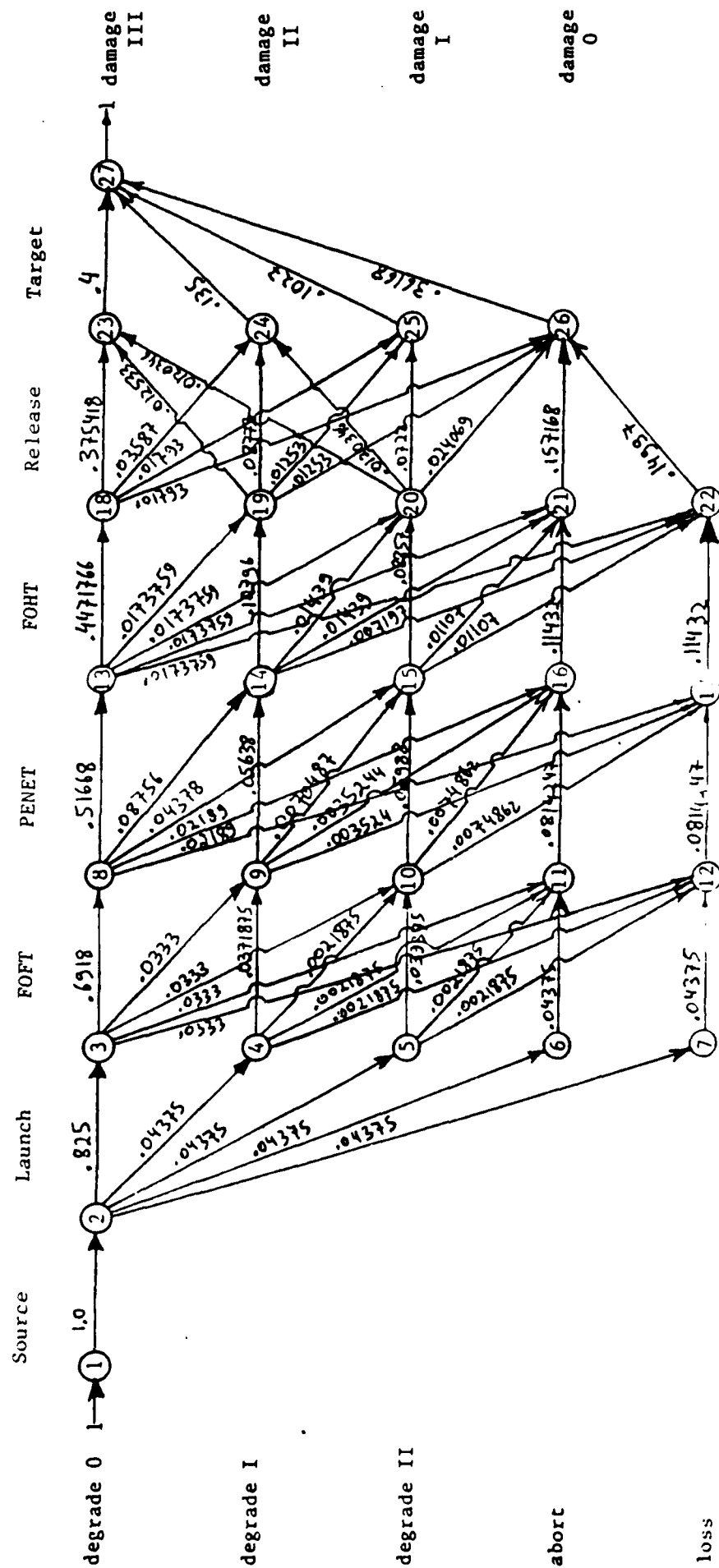


FIGURE 7  
CONSTRAINED PROPORTIONAL ALLOCATION



OPTIMAL OBJECTIVE = 25.095

## 6. Conclusions: Extensions and Further Uses

The above example, although entirely hypothetical, is nevertheless illustrative of the kinds of uses to which these stochastic network models can be put in, say, analyzing and evaluating possible design changes for missions that might be typical for a B52G bomber. Of course, actual applications would involve much more complex network representations and data arrays but the simple example of Table 1 at least serves to exhibit some of the complexities that need to be considered.

Of course we have not exploited all aspects of what is available from these solutions. The added advantage that is available from the associated dual variables makes it possible to assess possible alternatives in the budgetary amounts by relating the effects on damage scores without any need for carrying through detailed solutions. Alternatively sensitivity analysis techniques are available for assessing the consequences of data variations in both costs and particular probabilities--again by reference to their effects on damage scores with additional details also available for review and assessment when wanted.

These stochastic networks may also be altered in a variety of ways. For instance, circumstances may need to be considered in which a specified goal (in terms of expected damage scores) must be attained "at any cost". Alternatively we might ask "what is the cost required to enhance the expected damage score to certain levels?".

Only a slight modification in the mathematical formulation is needed to answer such questions. By switching the roles of the objective and the first constraint in the model ((1) and (2) above), eliminating the budget parameter  $B$  and introducing a goal parameter  $g$ , we get:

$$(9) \quad \text{Min } \sum_{k \in M} C_k(y_k)$$

s.t.

$$(10) \quad \sum_{k \in W} w_k x_k \geq g \quad (\text{goal constraint})$$

(3) - (8) unchanged.

Both the above formulation and the initial one were achieved as convex programming problems in order to provide the ability to compute and analyze constructively systems involving many subsystems and states. This achievement makes it possible to utilize the powerful capabilities of linear programming in methods like "successive linear programming" or other separable programming techniques without requiring the development of new algorithms for non-convex programming problems. We contrast our work with the stochastic network reliability efforts in the literature, cf. [10], wherein only special networks with very special prescriptions on reliabilities, e.g. all equal, are considered and with attempts to determine the probability to successfully (or the obverse, failure to) transit the network. These are in terms of special algorithms which are NP-complete (thereby presumably impossible for large networks), hence for which heuristics are hopefully offered with little guarantee of closeness to exact solutions.

As mentioned initially also, we have limited ourselves in this paper to modeling the case of a single aircraft system. Multiple aircraft systems are, of course, still more complicated but can be handled by extensions, which we shall produce in subsequent work, of the basic formulations herein.

In the above examples the objectives are oriented toward maximizing the objective damage scores. Other objectives are also possible. If estimates of probabilities for various kinds of enemy countermeasures are available,



for instance, the objective might be reorientation toward maximizing the minimum probability of achieving certain damage scores.

Still other possibilities are present, of course, and so are alternate uses. As noted at the outset of this paper the Air Force uses a variety of simulation models and modes of analysis. The network representations used here may also be used in that mode, if desired, and additionally it may be tied into these other planning instruments when tie-backs are to be secured to parts replenishment and other facets of inventory planning. Finally, and perhaps most importantly, these network concepts and models provide for easy review and understanding by managers as well as analysts and hence can be used as a source of guidance for planning data system and related aids to decision making.

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